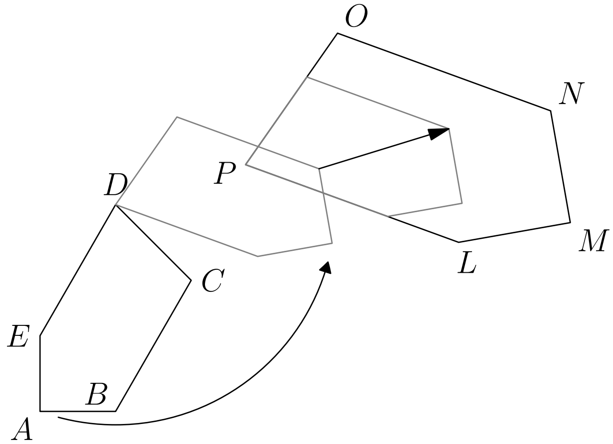
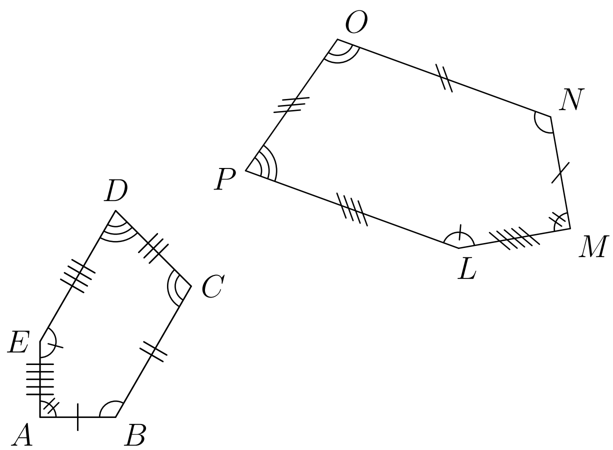
**Summary**

# 1 What is Similarity and Why Should I Care?

Two figures are **similar** if it is possible to move one onto the other by a sequence of shifts, rotations, flips, and/or *scalings* so that if one figure lies on top of the other figure, and they match exactly.

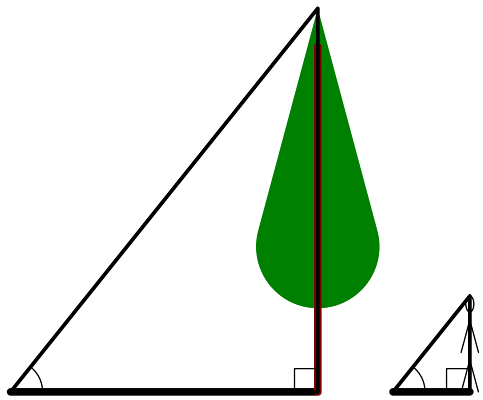



For two polygons this is the same as saying that corresponding sides are proportional and corresponding angles are equal.




$$ABCDE \sim MNOPL.$$

We can use similar triangles to measure objects that we can't measure directly, such as trees. Most often we find two triangles that have equal angles (creating a triangle if we have to), use AA similarity, and set up a proportion.



To find the lengths in a similar figure,

- first find the similarity ratio, and
- then multiply each length by this ratio.

If we scale a figure by a constant $k$, then the perimeter scales by the same constant $k$.

As with congruent triangles, be careful about matching up corresponding vertices.

## 2 How to Show Triangles are Similar

The criteria for similarity is very similar to the criteria for congruence, except that we need the sides to be proportional, rather than equal. For instance in SAS congruence we need

$$XY = AB, \qquad YZ = BC, \qquad \angle XYZ = \angle ABC,$$

but for SAS congruence we need

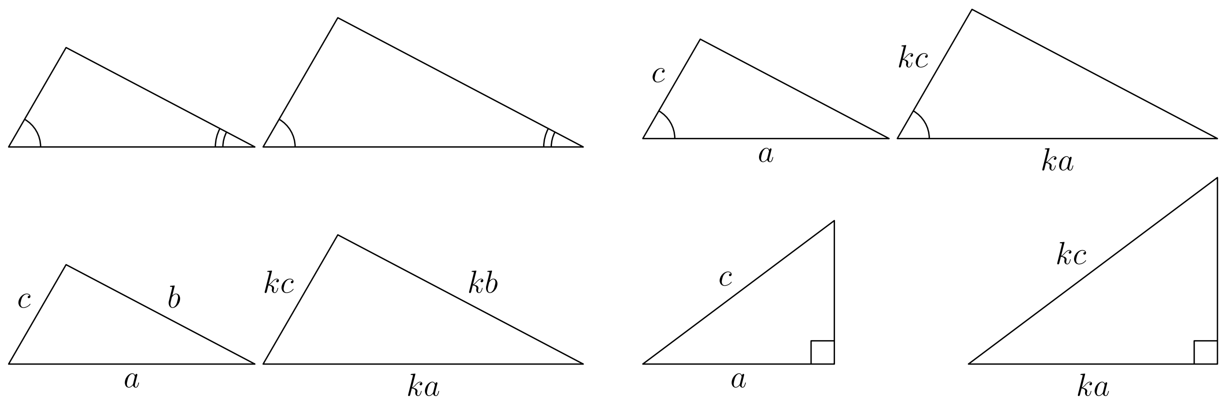
$$\frac{XY}{AB} = \frac{YZ}{BC} = k, \qquad \angle XYZ = \angle ABC.$$

| Congruence criteria | Similarity criteria |
|---|---|
| AAS/ASA | AA |
| SAS | SAS |
| SSS | SSS |
| HL | HL |

Note that the only criterion which is named differently is AA. However, it still follows the pattern of “just replace an equality with a proportion.” This is since the single equality $AB = XY$ becomes a ratio $\frac{XY}{AB} = k$ which is allowed to be anything!

Use one of the following criteria to show two triangles are similar.

- AA: Two angles are equal.
- SAS: Two sides are in proportion and the angles in between are equal.
- SSS: All three corresponding sides are in proportion.
- HL: The two triangles are right triangles, and the hypotenuse and one of the legs are in proportion.

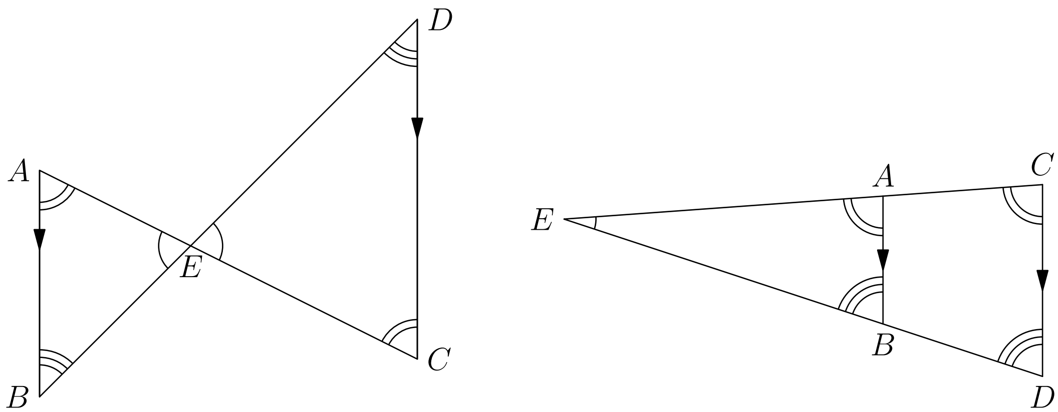



## 3 Common Similar Triangle Situations

The most common ways to get similar triangles are with *parallel lines* and *altitudes in right triangles*.

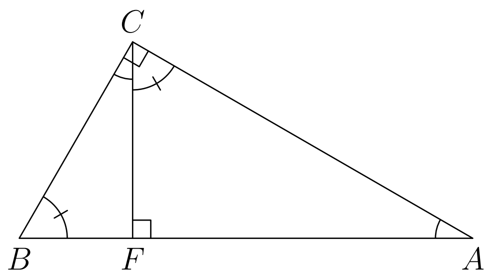
Suppose $\overline{AB} \parallel \overline{CD}$ and $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ intersect at $E$. Then

$$\triangle EAB \sim \triangle ECD.$$




$$\triangle EAB \sim \triangle ECD.$$

An altitude dropped from the vertex of a right triangle to the hypotenuse gives 3 similar right triangles. Each pair of triangles share a vertex in addition to a $90°$ angle.

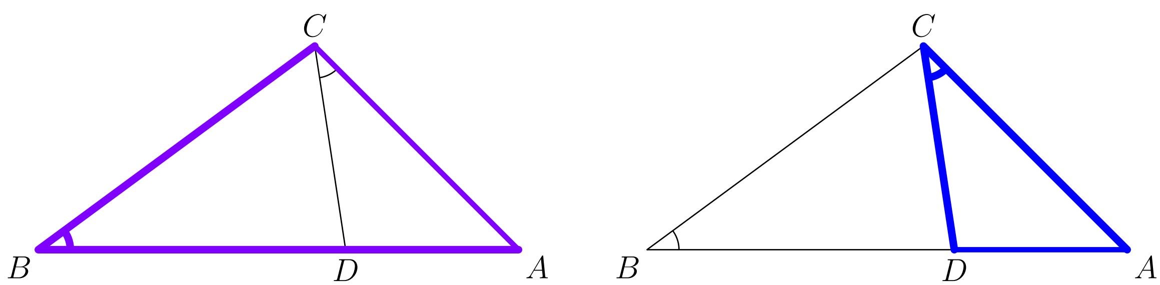



$$\triangle ABC \sim \triangle ACF \sim \triangle CBF.$$

The picture with right triangles is a special case of the following concept:

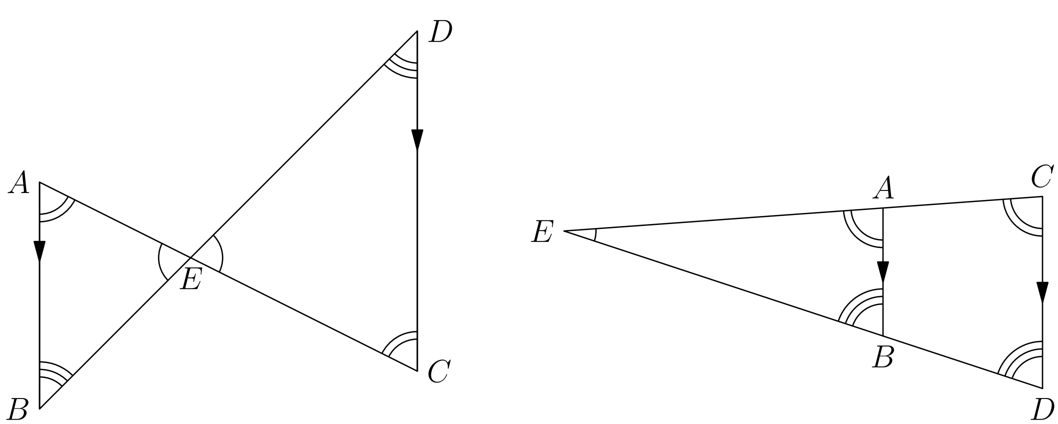
Similar triangles may overlap, and share angles and sides. Especially look for similar triangles that share angles, because this gives a free angle for AA similarity.

Be extra careful in writing the proportions when the triangles overlap! When in doubt, work from the similarity statement.




$$\triangle ABC \sim \triangle ACD$$

For similar triangles cut by parallel lines, we get more than just proportions coming from similar triangles, using the part-to-whole principle.



$$\frac{EA}{EB} = \frac{EC}{ED} = \frac{AC}{BD}.$$

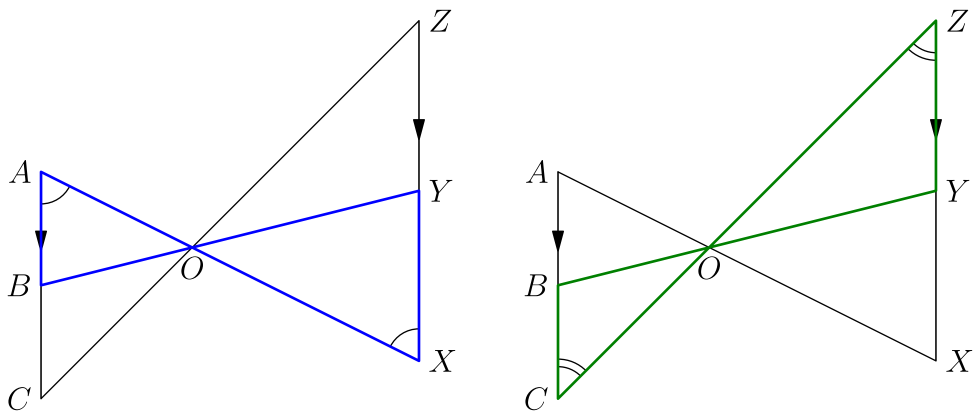
Note $\frac{AC}{BD} = \frac{EC+EA}{ED+EB} = \frac{EC-EA}{ED-EB}$.

Be careful about matching up sides: $\frac{EA}{ED} = \frac{EB}{EC}$ is incorrect, as is $\frac{EA}{AC} = \frac{AB}{CD}$. These are common mistakes in the left and right-hand diagrams, respectively.

## 4 Problem-Solving Strategies

Like any geometry problem, problems involving similar triangles may require multiple steps.

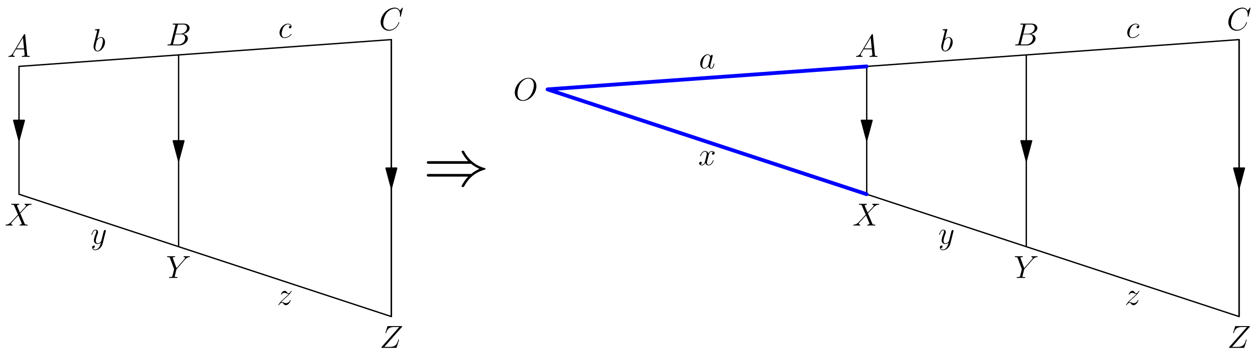
You may have to use more than one pair of similar triangles. To get ratios involving sides of different triangles, you may have to multiply ratios you get from different pairs of similar triangles.



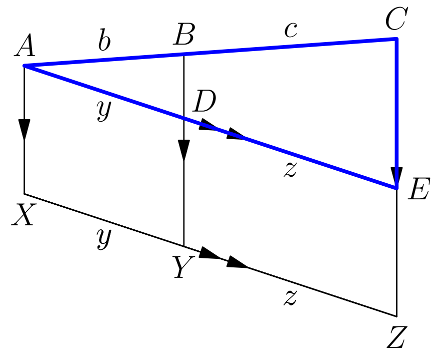

$$\frac{AB}{BC} = \frac{XY}{YZ}$$

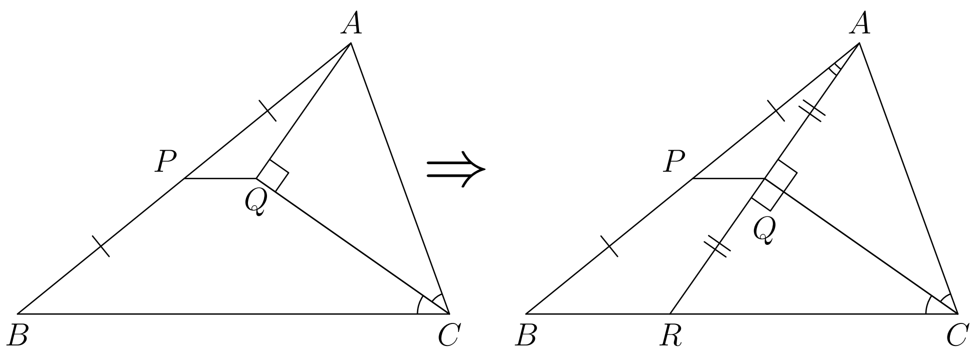
We can often create information (for instance, get similar triangles that aren't in the diagram) as follows:

- Extending lines until they intersect.
- Drawing a parallel line.
- Extending a segment that seems to end abruptly.

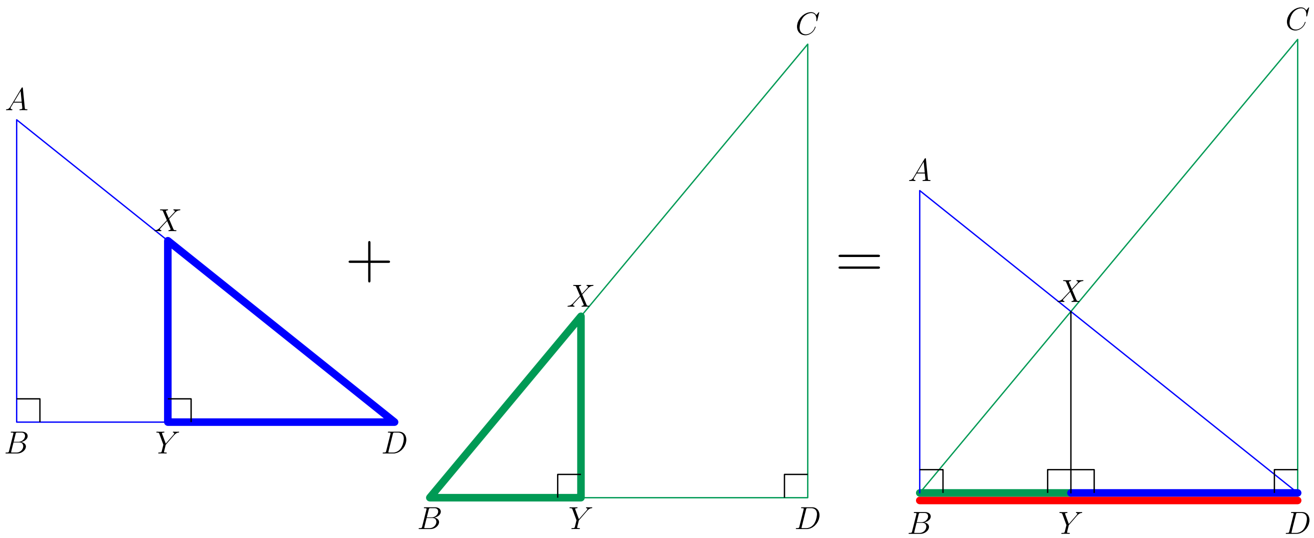



or



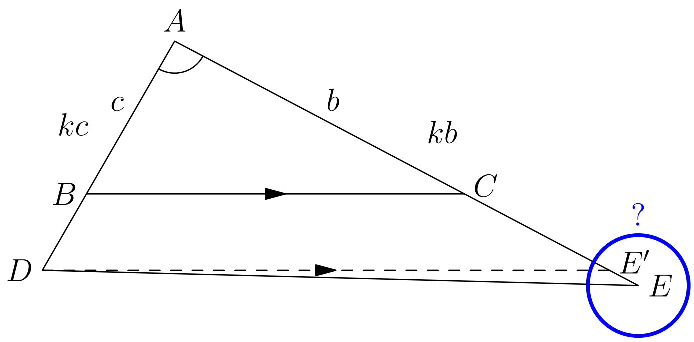

When you're stuck on a problem, ask yourself, *what information have I not used?* Think about how different parts of the problem fit together, and what constraints they create.




$$\frac{XY}{CD} + \frac{XY}{AB} = \frac{BD}{BD} = 1$$

In the proof of AA similarity, we encountered the following useful method of proof.

**Point redefinition:** Define a point that has some desired property, and show it coincides with another point in the diagram.




## 5 What Now? The Bigger Picture

Mathematicians are always on the lookout for ways to generalize facts they already know, to make them more powerful. This is exactly what we did in this chapter.

By *generalizing* the idea of **congruent** triangles using proportions instead of equality, you were able to construct a theory of **similar** triangles that...

- encompasses all of the theory of congruent triangles and more,
- is every bit as rigorous as the theory of congruent triangles, and
- offers a great way to calculate lengths in the real world.

(Proportions) + (Theory of congruent triangles) = (Theory of similar triangles)
(Eric) + (Connie) = (Knowing how to measure trees, <u>and</u> why it works).

Similar triangles will appear everywhere in geometry: for instance, when you learn about circles, you will see that a certain configuration of similar triangles always leads to circles, and that circles always give rise to similar triangles! As you saw, similar triangles appear when we have right triangles, and you will soon discover that similar triangles always lets us find missing lengths and angles in right triangles – this is the beginning of the branch of math called *trigonometry*. You've seen how rescaling a figure changes its perimeter, and soon discover how rescaling changes its *area* and *volume*. Finally, we've been talking about "shifts, rotations, reflections, and scalings"; with similar triangles you will soon be able to put the idea of *geometric transformations* on a firm footing.